Printed Page:- 04 **Subject Code:- AAS0104**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION (2023 - 2024)

Subject: Mathematical Foundations-I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

<u>SECTION-A</u> 20

1. Attempt all parts:-

1-a. The system of equations $x+2y+3z=1, 2x+y+3z=2, 5x+5y+9z=4$ has (CO1) 1

(a) only one solution

(b) infinitely many solution

(c) no solution

(d) none of these

1-b. Inverse of unitary matrix is a (CO1) 1

(a) symmetric matrix

(b) skew-symmetric matrix

(c) unitary matrix

(d) None of these

1-c. An inner product space from V x V into F which is called non-negativity if (C02) 1

(a) $(\alpha,\alpha)\geq 0 \; and \; (\alpha,\alpha)=1 \; \alpha\to 0$

(b) $(\alpha,\alpha)\geq 0 \; and \; (\alpha,\alpha)=0 \; \alpha\to 0$

(c) $(\alpha,\alpha)\leq 0 \; and \; (\alpha,\alpha)=1 \; \alpha\to 0$

(d) None of these

1-d. The vectors (x_1, x_2) and (y_1, y_2) are linearly dependent if and only if (CO2) 1

(a) $x_1x_2-y_1y_2=0$

(b) $y_1x_2-x_1y_2=0$

(c) $x_1x_2+y_1y_2$

(d) $x_1y_2-y_1x_2$

1-e. The $(n+1)^{th}$ derivative of x^n is (CO3) 1

(a) nx^{n+1}

(b) $(n+1)x^{n+1}$

(c) $(n+1)!\, x^{n+1}$

(d) None of these

1-f. If $y=\sin(m\sin^{-1}x)$ then after two time differentiation we get (CO3) 1

(a) $(1-x^2)y_2-xy_1-m^2y=0$

(b) $(1-x^2)y_2-xy_1+m^2y=0$

(c) $(1-x^2)y_2+xy_1+m^2y=0$

(d) None of these

1-g. If $\mathbf{x = u(1+v), y=v(1+u)}$, **then find the value of** $\frac{\partial(x,y)}{\partial(u,v)}$. (CO4) 1

(a) 1+u

(b) 1+u+v

(c) 1+v

(d) None of these

1-h. An error of 2% is made in measuring length and breadth then the percentage error in
the area of the rectangle is (CO4) 1

(a) 6

(b) 4

(c) 8

(d) 16

1-i. The average of all prime numbers between 30 and 50 is (CO5) 1

(a) 38.8

(b) 39.8

(c) 40.8

(d) None of these

1-j. A got 37.5 % marks less than B, then by what percent the marks of B is more than the marks of A? (CO5) 1

(a) 0.6

(b) 0.375

(c) 0.2727

(d) None of these

2. Attempt all parts:-

2.a. For the given matrix $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, prove that $A^3 = 19A + 30I$. (CO1) 2

2.b. Define the term Nullity for a linear transformation. (CO2) 2

2.c. Find the nth differential coefficients of $x^2 e^x$ (CO3) 2

2.d. Find the minimum value of $x^2 + y^2 + 6x + 12$. (CO4) 2

2.e. In a club, the average age of the members is 30 years, the average age of male members is 34 years and that of the female members is 26 years. Then the percentage of the male members ? (CO5) 2

SECTION-B 30

3. Answer any <u>five</u> of the following:-

3-a. Find the inverse of matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by using the elementary row transformation. (CO1) 6

3-b. Find all values of μ for which rank of the matrix $\begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to 3. (CO1) 6

3-c. Determine whether or not the following vectors form a basis of R^3 : (1, 1, 2), (1, 2, 5), (5, 3, 4). (CO2) 6

3-d. If α and β are vector in real vector space and if α + β is orthogonal to α - β then prove that $\|\alpha\| = \|\beta\|$. (CO2) 6

3.e. Find the nth derivative of $y = \sin x . \cos 3x$. (CO3) 6

3.f. If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_1 x_3}{x_2}$ and $y_3 = \frac{x_1 x_2}{x_3}$ then find the value of $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$. (CO4) 6

3.g. In a certain code language, 'no more food' is written as 'ta ka da' and 'more than that' is written as 'sa pa ka'. How is 'that' written in that code language? (CO5) 6

SECTION-C 50

4. Answer any <u>one</u> of the following:-

4-a. Determine the value of λ and μ that the equations
$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$
have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.(CO1) 10

4-b. Find the rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Prove that the Set of all solution (a, b, c) of the equation a + b + 2c = 0 is a subspace of the vector space $V_3(R)$. (CO2) 10

5-b. In V_2 (F) define for $\alpha = (a_1, a_2)$ and $\beta = (b_1, b_2)$, $\langle \alpha, \beta \rangle = 2a_1\overline{b_1} + a_1\overline{b_2} + a_2\overline{b_1} + a_2\overline{b_2}$. Show that this defines an inner product space on V_2(F). (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. If $x^x y^y z^z = c$, show that $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$. (CO3) 10

6-b. If u = f(y-z, z-x, x-y), prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Expand the function $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of (1, 1) upto and inclusive of second degree terms. Hence compute the value of the function at (1.1, 0.9) approximately. (CO4) 10

7-b. Find the extreme values of the function $x^3 + y^3 - 3xy$. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) The average of 5 consecutive numbers is n. If the next two numbers are also included, the average of the 7 number will ? (CO5) 10

(ii) A retailer marks all his goods at 50% above the cost price and offers a discount of 25% on the marked price. What is his actual profit on the sales? (CO5)

(iii) If A, B, C is 3 students. A got 20% more marks than B and 30 % less than C. if B got 175, the how much C got? (CO5)

8-b. (i) If the radius of the cylinder increases by 10 % and the height increases by 20%. Then, what is the change in the volume of the cylinder ?(CO5) 10

(ii) The average age of eight teachers in a school is 40 years. A teacher among them died at the age of 55 years whereas another teacher whose age was 39 years joins them. The average age of the teachers in the school now is (in years)? (CO5)

(iii) A machine is sold for Rs5060 at a gain of 10%. What would have been the gain or loss % if it had been sold for Rs 4370? (CO5)